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Gain Normalization of Lifted Filter Banks

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Abstract

The DC and Nyquist responses of the filters in a two-channel perfect reconstruction filter bank are expressed in terms of the lifting filters in a lifting decomposition. The computation makes use of the cascade-form representation of lifting steps as lower- and upper-triangular factor matrices in the polyphase-with-advance representation. A functional relationship is derived connecting the DC and Nyquist responses via the polyphase determinant, and it is shown that the responses for a lifted filter bank can be computed recursively using the DC responses of the lifting filters. These results are applied to derive the filter bank normalization specifications in Part 2 of the ISO/IEC JPEG 2000 still image coding standard.

Key words: Filter bank, wavelet, polyphase, lifting, JPEG 2000.

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1 Introduction

The filter banks considered in this paper have the form of Figure 1; see [1–5]. This direct-form representation is equivalent to the *polyphase-with-advance representation* in Figure 2 [6,7]. We study a cascade decomposition of these polyphase matrices known as the *lifting factorization* [8,6]. As proven in [6], lifting is a universal representation for two-channel finite impulse response (FIR) perfect reconstruction filter banks. Figure 3 shows a lifting decomposition of an analysis filter bank with two cascaded lifting filters, $S_0(z)$ and $S_1(z)$. The *scaling factor*, K, specifies a gain normalization of the channels.

Since lifting is a universal representation, arbitrary filter banks can be specified in terms of a lifting factorization; i.e., a sequence of lifting filters, $S_i(z)$. When a filter bank is defined in this way, the DC and Nyquist responses are determined by the lifting filters and the scaling factor, K. Thus, any effort to scale the channel gains must compute the contribution attributable to the lifting filters. This paper derives a method of performing such a calculation via a recursion on the lifting filters, without requiring conversion of the lifting representation into the direct-form representation of Figure 1.

1.1 Motivation and Outline of the Paper

The need for such analysis arose during the authors' involvement in the ISO/IEC JPEG 2000 still image coding standard. While JPEG 2000 Part 1 (the "Baseline" standard) [9] only provides two predefined filter banks, users may specify arbitrary two-channel FIR filter banks in JPEG 2000 Part 2 (Extensions) [10] (henceforth just "Part 2"). Part 2 enables this by providing syntax for compressed codestreams that allows users to signal the lifting parameters of a custom filter bank.

To simplify the specification of quantization and entropy coding, however, gain normalization is not left up to the user, and the standard specifies normalization requirements that filter banks must satisfy in JPEG 2000 codestreams. These requirements, which total less than one page of content in Part 2 [10], are not derived or explained in the standard and are extremely cryptic as written. Our goal in the present paper is to construct a lifting-domain approach to calculating a filter bank's DC and Nyquist responses that can be applied to deriving and explaining the normalization specifications in JPEG 2000 Part 2. Gain normalization is a fundamental part of digital filtering, and this paper should also be of benefit to anyone developing other signal processing applications using lifted filter banks.

Readers desiring more background on JPEG 2000 are referred to [11–16]. A great deal of useful material on filter banks and lifting is provided in the book of Taubman and Marcellin [16], but that volume principally addresses JPEG 2000 Part 1, and Part 2 extensions are only mentioned briefly. In particular, the normalization specifications for Part 2 user-defined filter banks are not covered in [16].

The outline of the paper is as follows. In the next subsection we summarize a few results from the polyphase-with-advance theory; readers who desire a more thorough treatment are directed to [7]. Section 2 explores the relationship between the DC and Nyquist responses in a perfect reconstruction filter bank and derives recursive formulas for filter responses in terms of the DC responses of the lifting filters, without requiring conversion to a direct-form representation. Section 3 reveals the functional intent of the JPEG 2000 filter bank normalization requirements and expresses them in terms of a recursive formula for lowpass DC response.

1.2 The Polyphase-with-Advance Representation

The relationship between the lowpass and highpass analysis filters, $H_0(z)$ and $H_1(z)$, in Figure 1 and the filter elements in the polyphase analysis matrix, $\mathbf{H}_a(z)$, in Figure 2 can be stated succinctly in matrix-vector arithmetic. Using an underscore to denote vectors, the relationship, which is derived in [7, Formula (9)], can be written

$$\underline{H}(z) \equiv \begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} = \mathbf{H}_a(z^2) \begin{bmatrix} 1 \\ z \end{bmatrix} . \tag{1}$$

Figure 2 is an FIR filter bank with FIR inverse if and only if $\det \mathbf{H}_a(z) = az^{-d}$ for some $a \neq 0$, $d \in \mathbb{Z}$. In [6], Daubechies and Sweldens simplify this condition, for the sake of defining lifting factorizations, by assuming that $a = 1$ and $d = 0$:

$$\det \mathbf{H}_a(z) = 1 . \tag{2}$$

They prove that any FIR filter bank satisfying (2) has a *lifting factorization*,

$$\mathbf{H}_a(z) = \operatorname{diag}(1/K,\, K)\, \mathbf{S}_{N_{LS}-1}(z) \cdots \mathbf{S}_1(z)\, \mathbf{S}_0(z) , \tag{3}$$

where the lifting matrices, $\mathbf{S}_i(z)$, are upper- or lower-triangular with ones on the diagonal and a lifting filter, $S_i(z)$, in the off-diagonal position. N_{LS} denotes the number of lifting steps. The scaling factor, K, constitutes a single degree of freedom that can be adjusted to redistribute gain between the lowpass and highpass channels. Note that (2), together with the upper- and lower-triangular structure of the lifting matrices, forces the scaling matrix, $\operatorname{diag}(1/K,\, K)$, to have a determinant of 1.

The *update characteristic*, m_0, of the first lifting step in Figure 3 is defined as "lowpass" (coded with a zero: $m_0 = 0$). Similarly, the update characteristic, m_1, of the next lifting step is "highpass" ($m_1 = 1$). Because the update characteristic of successive steps alternates, it is only necessary to transmit to the decoder the update characteristic, m_{init}, of the initial synthesis lifting step, which is the same as the update characteristic of the *last* analysis lifting step, $\mathbf{S}_{N_{LS}-1}(z)$.

Figure 4 is a *reversible* filter bank implementation: the rounding function, R, in each lifting step implies that Figure 4 maps integer input to integer subbands. Moreover, the impulse response coefficients of the lifting filters in a reversible system are restricted to dyadic rationals. This means that a reversible system is capable of bit-perfect reconstruction of integer signals in fixed-precision arithmetic, which enables lossless subband coding. The system in Figure 3 is referred to as *irreversible* to emphasize that it is not immune to roundoff error or other finite-precision effects; irreversible filter banks are only mathematically invertible in “perfect” arithmetic.

2 Calculation of Responses for Lifted Filter Banks

When designing a digital filter, one degree of freedom is usually regarded as trivial; e.g., the DC response, $H_0(1)$, of a lowpass filter, or the Nyquist response, $H_1(-1)$, of a highpass filter. In the case of filter *banks* satisfying (2), one degree of freedom is consumed by normalizing the polyphase determinant. This means we cannot further normalize both the lowpass DC response and the highpass Nyquist response without using up a nontrivial degree of freedom. In this section we derive the relationship between the DC and Nyquist responses in a perfect reconstruction filter bank and express the lowpass DC response explicitly in terms of lifting filters.

2.1 Relationship Between DC and Nyquist Responses

Formula (1) allows us to represent both the DC and Nyquist responses for the filter bank by a single matrix equation,

$$\begin{bmatrix} H_0(1) & H_0(-1) \\ H_1(1) & H_1(-1) \end{bmatrix} = \begin{bmatrix} H_{a_{00}}(1) & H_{a_{01}}(1) \\ H_{a_{10}}(1) & H_{a_{11}}(1) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} . \tag{4}$$

Taking the determinant of both sides of (4) gives

$$H_0(1)\, H_1(-1) - H_1(1)\, H_0(-1) = -2 \det \mathbf{H}_a(1) . \tag{5}$$

(This equation is also derived, by somewhat different methods and in different notation, in [16, Section 6.1.3].) If the filter bank satisfies one or more vanishing moment conditions (e.g., if it is a wavelet filter bank), then the second term on the left-hand side of (5) vanishes, $H_1(1)H_0(-1) = 0$, leaving a relationship connecting the lowpass DC response and the highpass Nyquist response:

$$H_0(1)\, H_1(-1) = -2 \det \mathbf{H}_a(1) . \tag{6}$$

If, in addition, $\det \mathbf{H}_a(z) = 1$ and $H_0(1) = 1$, then the highpass Nyquist response is

$$H_1(-1) = -2\ . \tag{7}$$

In practice, systems like JPEG 2000 that use multiresolution filter bank cascades usually use wavelet filter banks because the regularity of the analog wavelet influences the properties of the cascaded filter bank. There are also approximation-theoretic reasons for using filter banks corresponding to wavelets with a couple of vanishing moments in source coding applications. If, for some reason, an application employs a filter bank that does *not* satisfy any vanishing moment conditions, then the second term in (5) will not be zero and $H_1(-1)$ will generally not equal -2.

2.2 Calculation of DC and Nyquist Responses

Assume a filter bank is specified by lifting matrices, $\mathbf{S}_i(z)$. We compute the contribution to the DC and Nyquist responses made by the lifting filters; in Section 3 we adjust the scaling factor, K, to achieve a desired lowpass DC response. Note that these formulas and the gains computed from the direct-form impulse responses are only equivalent in perfect arithmetic; i.e., without regard for finite-precision effects. Let $\mathbf{E}(z)$ denote the *unnormalized* cascade of lifting steps,

$$\mathbf{E}(z) \equiv \mathbf{S}_{N_{LS}-1}(z) \cdots \mathbf{S}_1(z)\mathbf{S}_0(z)\ ,$$

and let $\mathbf{E}^{(n)}(z)$ denote the n^{th} partial product:

$$\mathbf{E}^{(n)}(z) = \mathbf{S}_n(z) \cdots \mathbf{S}_0(z) \quad \text{for} \quad n = 0, \ldots, N_{LS} - 1.$$

The vector of corresponding lowpass and highpass scalar filters is given by (1):

$$\begin{aligned} \begin{bmatrix} E_0^{(n)}(z) \\ E_1^{(n)}(z) \end{bmatrix} &\equiv \underline{E}^{(n)}(z) = \mathbf{S}_n(z^2) \cdots \mathbf{S}_0(z^2) \begin{bmatrix} 1 \\ z \end{bmatrix} \\ &= \mathbf{S}_n(z^2)\, \underline{E}^{(n-1)}(z)\ . \end{aligned} \tag{8}$$

Using (8), the DC responses can be built from the lifting steps via the recursion

$$\underline{E}^{(n)}(1) = \mathbf{S}_n(1)\, \underline{E}^{(n-1)}(1) \quad \text{for } n = 0, \ldots, N_{LS} - 1, \text{ where} \tag{9}$$

$$\underline{E}^{(-1)}(1) \equiv \begin{bmatrix} 1 \\ 1 \end{bmatrix} .$$

Similarly, the vector of Nyquist responses is given by the recursion

$$\underline{E}^{(n)}(-1) = \mathbf{S}_n(1)\, \underline{E}^{(n-1)}(-1) \quad \text{for } n = 0, \ldots, N_{LS} - 1, \text{ where} \tag{10}$$

$$\underline{E}^{(-1)}(-1) \equiv \begin{bmatrix} 1 \\ -1 \end{bmatrix} .$$

These formulas raise an interesting asymmetry between the DC and Nyquist responses of the lifting filters. By (4), both the DC and Nyquist responses of the analysis filters, $H_0(z)$ and $H_1(z)$, are determined by the DC responses of the polyphase filters, $H_{a_{ij}}(1)$, while (9) and (10) show that the DC and Nyquist responses of the analysis filters are also determined by the DC responses of the lifting filters, $S_n(1)$.

To see that we cannot compute the responses of the analysis filters in terms of the *Nyquist* responses of the lifting filters, consider banks of WS (*whole-sample symmetric*, or type I) linear phase filters. As shown in [16,7], WS filter banks always have lifting factorizations in terms of HS (*half-sample symmetric*, or type II) linear phase lifting filters, which have a Nyquist response of zero: $S_i(-1) = 0$. Note that different (unnormalized) lifted WS filter banks can have different responses at DC or Nyquist; e.g., this is the case with the 5-tap/3-tap and 9-tap/7-tap WS wavelet filter banks specified in JPEG 2000 Part 1 [9, Annex F]. Thus, any functional expression that depends only on the Nyquist responses of the lifting filters is incapable of computing the DC or Nyquist responses of WS analysis filters, $H_0(z)$ and $H_1(z)$.

An even more interesting situation holds for filter banks containing an HS lowpass filter and an HA (*half-sample antisymmetric*, or type IV) highpass filter. As shown in [7], HS/HA filter banks can always be lifted from a concentric, equal-length "base" HS/HA filter bank by lifting steps with WA (*whole-sample antisymmetric*, or type III) linear phase lifting filters. For instance, a 2-tap/10-tap HS/HA filter bank [10, Annex H.4.1.1.3] can be lifted from the Haar filter bank by a fourth-order WA highpass lifting update. WA filters have a DC response of zero, $S_i(1) = 0$, and are therefore subject to the following consequence of formulas (9) and (10).

Theorem 1 *Given a base filter bank,* $\mathbf{B}(z)$*, all filter banks lifted from* $\mathbf{B}(z)$ *using lifting filters with a DC response of zero necessarily have the same DC and Nyquist responses as* $\mathbf{B}(z)$*.*

Theorem 1 implies that, e.g., the 2-tap/10-tap HS/HA filter bank has the same DC and Nyquist responses as the Haar filter bank. In comparison, there is no analog of Theorem 1 for WS filter banks since there are no constraints on the DC responses of their HS lifting filters.

2.3 Recursive Calculation of the Lowpass DC Response

In light of Section 2.1, we focus on normalizing the lowpass DC response in a filter bank using (9). If (2) is not assumed then the highpass Nyquist response can be treated as a second trivial degree of freedom and normalized using (10).

We will compute $E_0(1) \equiv E_0^{(N_{LS}-1)}(1)$ by deriving a scalar recursion, based on (9), that expresses $E_0(1)$ in terms of the DC responses, $S_i(1)$, of the lifting filters. Let B_n denote the *most recently modified entry* in the DC response vector, $\underline{E}^{(n)}(1)$. For instance, if $\mathbf{S}_0(z)$ is upper-triangular then the parameter B_0 is defined as follows:

$$\begin{bmatrix} E_0^{(0)}(1) \\ E_1^{(0)}(1) \end{bmatrix} = \begin{bmatrix} 1 & S_0(1) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} S_0(1)+1 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} B_0 \\ 1 \end{bmatrix} .$$

The last equation above defines $B_0 = S_0(1) + 1$. Using (9) to define $B_1, B_2, \ldots$:

$$\begin{bmatrix} E_0^{(1)}(1) \\ E_1^{(1)}(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ S_1(1) & 1 \end{bmatrix} \begin{bmatrix} B_0 \\ 1 \end{bmatrix} = \begin{bmatrix} B_0 \\ S_1(1)\,B_0+1 \end{bmatrix} \equiv \begin{bmatrix} B_0 \\ B_1 \end{bmatrix} ,$$

which defines $B_1 = S_1(1)B_0 + 1$, followed by

$$\begin{bmatrix} E_0^{(2)}(1) \\ E_1^{(2)}(1) \end{bmatrix} = \begin{bmatrix} 1 & S_2(1) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} B_0 \\ B_1 \end{bmatrix} = \begin{bmatrix} S_2(1)\,B_1+B_0 \\ B_1 \end{bmatrix} \equiv \begin{bmatrix} B_2 \\ B_1 \end{bmatrix} ,$$

which defines $B_2 = S_2(1)B_1 + B_0$, etc. Analogous definitions hold for B_0, B_1, etc. if the first lifting step is lower-triangular:

$$\begin{bmatrix} E_0^{(0)}(1) \\ E_1^{(0)}(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ S_0(1) & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ S_0(1)+1 \end{bmatrix} \equiv \begin{bmatrix} 1 \\ B_0 \end{bmatrix} ,$$

which defines $B_0 = S_0(1) + 1$, followed by

$$\begin{bmatrix} E_0^{(1)}(1) \\ E_1^{(1)}(1) \end{bmatrix} = \begin{bmatrix} 1 & S_1(1) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ B_0 \end{bmatrix}$$

$$= \begin{bmatrix} S_1(1)\, B_0 + 1 \\ B_0 \end{bmatrix} \equiv \begin{bmatrix} B_1 \\ B_0 \end{bmatrix}$$

and so forth. In either case, the matrix-vector recursion (9) can be written very compactly as a scalar recursion on B_n:

$$B_n = S_n(1)\, B_{n-1} + B_{n-2} \quad \text{for} \quad n = 0, \ldots, N_{LS} - 1, \quad \text{where} \tag{11}$$

$$B_{-1} \equiv B_{-2} \equiv 1 \, .$$

The question of which parameter, B_n, gives the value of the final lowpass DC response, $E_0(1)$, depends on the update characteristic, m_{init}, of the last analysis lifting step. If $m_{init} = 0$, i.e., if $\mathbf{S}_{N_{LS}-1}(z)$ is a lowpass lifting step (upper-triangular matrix), then the last step in the recursion (11) updates the lowpass DC response so

$$E_0(1) = B_{N_{LS}-1} \quad \text{if} \quad m_{init} = 0 \, . \tag{12}$$

If $m_{init} = 1$, i.e., if $\mathbf{S}_{N_{LS}-1}(z)$ is a *highpass* step (lower-triangular matrix), then the last step in the recursion (11) updates the *highpass* DC response: $B_{N_{LS}-1} = E_1(1)$. Thus, the final lowpass DC response is given by the next-to-last step in the recursion:

$$E_0(1) = B_{N_{LS}-2} \quad \text{if} \quad m_{init} = 1 \, . \tag{13}$$

3 Normalization of User-Defined Filter Banks in JPEG 2000

The normalization requirements for user-defined filter banks in JPEG 2000 Part 2 form one of the more inscrutable specifications in the standard. In particular, the signal processing intent is not at all obvious from the cryptic specifications (which the authors helped write) in Part 2 [10, Annex G.2.1 and Annex H.1.1]. This is partly because the standard documents are written in "standardese" that avoids most higher-level concepts from digital signal processing, such as matrix-vector algebra or transform analysis of linear filters. Instead, all specifications are written as explicit arithmetic formulas in order to avoid normative references to external literature and to minimize the terminology defined normatively within the standard. This makes both writing and reading such a document extremely challenging, and writing the filter bank normalization requirements gave the standards committee enough trouble that the authors believe a high-level derivation will prove useful.

Given an irreversible, lifted analysis filter bank satisfying (2),

$$\mathbf{H}_a(z) = \mathrm{diag}(1/K,\ K)\,\mathbf{E}(z)\ ,$$

the intent of the normalization specifications in JPEG 2000 Part 2 [10, Annex G.2.1] is to set the scaling factor, K, so that the lowpass analysis filter will have unit response at DC, $H_0(1) = 1$. To achieve this, K must equal the DC response of the *unnormalized* lowpass filter corresponding to $\mathbf{E}(z)$; i.e.,

$$K = E_0(1) \neq 0\ . \tag{14}$$

Unfortunately, the JPEG 2000 standard avoids saying so in plain language. Instead of simply *defining* K in terms of a formula like (14), Part 2 lets the encoder signal a value for K in the JPEG 2000 codestream and then specifies a "normalization" the codestream must satisfy in terms of the arcane scalar recursion (11). (This particular scalar recursion is used in the standard to avoid reliance on matrix-vector notation.) We now show how to interpret the normalization specifications given in [10, Annex G.2.1] in terms of the theory developed in Section 2.

3.1 Filter Bank Normalization Specifications in JPEG 2000 Part 2

JPEG 2000 Part 1 provides two linear phase filter bank options. One is a reversible, multiplier-free lifted implementation of the 5-tap/3-tap piecewise-linear B-spline wavelet filter bank constructed by LeGall and Tabatabai [17]. The other option is the irreversible 9-tap/7-tap floating-point wavelet filter bank constructed by Cohen, Daubechies, and Feauveau [18,1]. Both are defined in such a way that $H_0(1) = 1$.

JPEG 2000 Part 2 allows lifted representations for arbitrary FIR filter banks. Part 2 Annex G specifies signaling and implementation for WS filter banks lifted using HS lifting filters, while Annex H covers *arbitrary* lifted FIR filter banks with no symmetry assumptions on either the analysis filters or the lifting filters. Part 2 uses D_i to denote the "sum of the lifting coefficients"; i.e., the DC response, $D_i \equiv S_i(1)$, of the i^{th} lifting filter. This puts (11) into the form of [10, Annex G.2.1, formula G.1]:

$$B_n = D_n\, B_{n-1} + B_{n-2} \quad \text{for} \quad n = 0, \ldots, N_{LS} - 1, \quad \text{where} \tag{15}$$

$$B_{-1} \equiv B_{-2} \equiv 1\ .$$

The analysis that led to formulas (12) and (13) now gives the normalization specifications for user-defined irreversible filter banks that appear in [10, Annex G.2.1.2]:

$$B_{N_{LS}-2} = K \quad \text{if } m_{init} = 1, \quad \text{or} \tag{16}$$

$$B_{N_{LS}-1} = K \quad \text{if } m_{init} = 0. \tag{17}$$

It might seem tortured logic to present these specifications (without explanation) as requirements that a codestream must satisfy. Why not simply *define* K in the decoder via (15), (16) and (17)? One reason is that it is more convenient to signal K (even though it is redundant) than to force the decoder to compute K using (15). Another is to avoid the finite-precision effects that would occur if K were computed by the decoder using (15). This makes it necessary to ensure that the codestream contains the correct scaling factor, a somewhat awkward requirement since the JPEG 2000 standard only imposes specifications on decoders and codestreams.

Finally, if the filter bank is *reversible* then there are no scaling factors. In this case, the standard requires that the lifting filters be scaled so that the *unnormalized* lowpass DC response is unity. Note that this does not depend on the rounding rules used in the reversible implementation; i.e., it specifies the lowpass DC gain of the *linear* filter bank. This is expressed in [10, Annex G.2.1.1] for reversible filter banks by

$$B_{N_{LS}-2} = 1 \quad \text{if } m_{init} = 1, \quad \text{or} \tag{18}$$
$$B_{N_{LS}-1} = 1 \quad \text{if } m_{init} = 0. \tag{19}$$

4 Conclusions

Formulas have been derived for the DC and Nyquist responses of a lifted two-channel filter bank in terms of the lifting filters, without requiring conversion to a direct-form representation. For wavelet filter banks, a functional relationship between the filters determines the highpass Nyquist response in terms of the lowpass DC response and the polyphase determinant. For arbitrary lifted filter banks, both the DC and Nyquist responses are functions of the DC responses of the lifting filters, and recursive formulas have been derived for computing the analysis filter responses. These results have been applied to derive the normalization specifications for filter banks in Part 2 of the ISO/IEC JPEG 2000 image coding standard.

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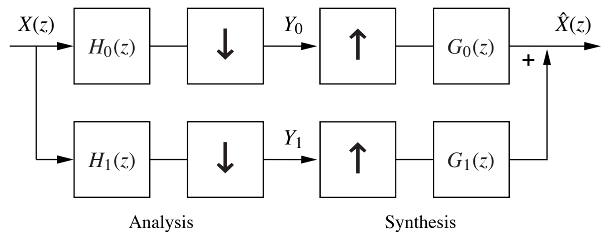

Fig. 1. Direct-form representation of a two-channel multirate filter bank.

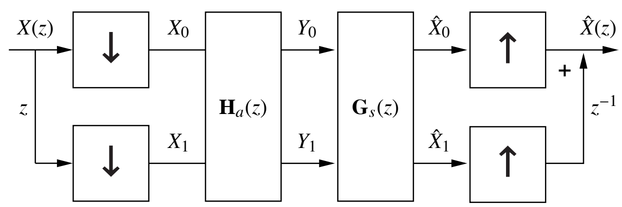

Fig. 2. The polyphase-with-advance filter bank representation.

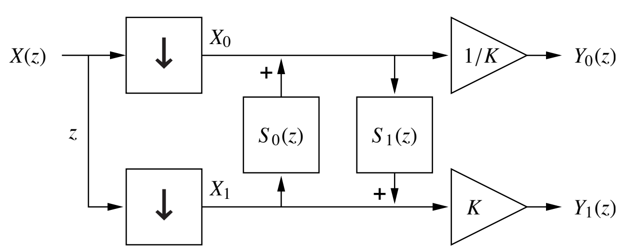

Fig. 3. Example of a lifted two-channel analysis filter bank.

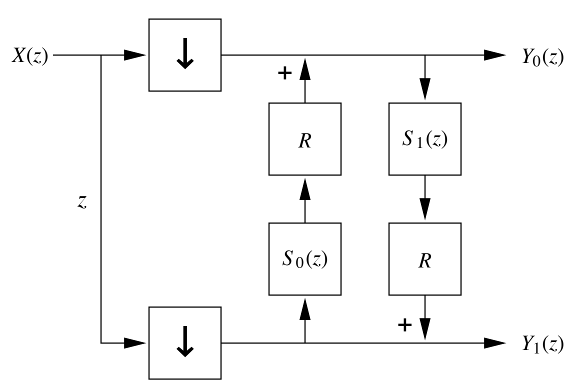

Fig. 4. Reversible implementation of a lifted analysis filter bank.